2020 AP CALCULUS BC FORMULA LIST

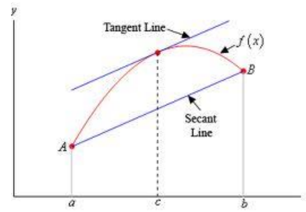
Definition of the derivative:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \qquad f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \text{ (Alternative form)}$$

Definition of continuity: f is continuous at c if and only if

1) $f(c)$ is defined;
2) $\lim_{x \to c} f(x)$ exists;
3) $\lim_{x \to c} f(x) = f(c)$.

Mean Value Theorem: If f is continuous on $[a, b]$ and differentiable on (a, b), then there exists a number c on (a, b) such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.



Intermediate Value Theorem: If f is continuous on $[a, b]$ and k is any number between $f(a)$ and $f(b)$, then there is at least one number c between a and b such that $f(c) = k$.

$$\frac{d}{dx}\left[x^n\right] = nx^{n-1} \qquad \frac{d}{dx}\left[\sqrt{u}\right] = \frac{du/dx}{2\sqrt{u}} \qquad \frac{d}{dx}\left[\frac{1}{x}\right] = -\frac{1}{x^2}$$

$$\frac{d}{dx}\left[f(x)g(x)\right] = f(x)g'(x) + g(x)f'(x) \qquad \frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{\left(g(x)\right)^2}$$

$$\frac{d}{dx}\left(f\left(g(x)\right)\right) = f'\left(g(x)\right) \cdot g'(x)$$

$$\frac{d}{dx}[\sin u] = \cos u \frac{du}{dx} \qquad \frac{d}{dx}[\cos u] = -\sin u \frac{du}{dx}$$

$$\frac{d}{dx}[\tan u] = \sec^2 u \frac{du}{dx} \qquad \frac{d}{dx}[\cot u] = -\csc^2 u \frac{du}{dx}$$

$$\frac{d}{dx}[\sec u] = \sec u \tan u \frac{du}{dx} \qquad \frac{d}{dx}[\csc u] = -\csc u \cot u \frac{du}{dx}$$

$$\frac{d}{dx}[\ln u] = \frac{1}{u} \frac{du}{dx} \qquad \frac{d}{dx}[\log_a u] = \frac{1}{u \ln a} \frac{du}{dx}$$

$$\frac{d}{dx}[e^u] = e^u \frac{du}{dx} \qquad \frac{d}{dx}[a^u] = a^u \ln a \frac{du}{dx}$$

$$\frac{d}{dx}[\arcsin u] = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx} \qquad \frac{d}{dx}[\arccos u] = -\frac{1}{\sqrt{1-u^2}} \frac{du}{dx}$$

$$\frac{d}{dx}[\arctan u] = \frac{1}{1+u^2} \frac{du}{dx} \qquad \frac{d}{dx}[\operatorname{arc}\cot u] = -\frac{1}{1+u^2} \frac{du}{dx}$$

Definition of a definite integral: $\displaystyle\int_a^b f(x)\,dx = \lim_{\Delta x \to 0} \sum_{k=1}^{n} f(x_k) \cdot (\Delta x_k) = \lim_{n \to \infty} \sum_{k=1}^{n} f(x_k) \cdot (\Delta x_k)$

$\int \cos u \, du = \sin u + C$

$\int \sin u \, du = -\cos u + C$

$\int x^n dx = \dfrac{x^{n+1}}{n+1} + C, \ n \neq -1$

$\int \dfrac{1}{u} du = \ln|u| + C$

$\int e^u du = e^u + C$

$\int a^u du = \dfrac{a^u}{\ln a} + C$

$\int f'(g(x)) \cdot g'(x) dx = f(g(x)) + C$

<u>Definition of a Critical Number</u>:
Let f be defined at c. If $f'(c) = 0$ or if f' is undefined at c, then c is a critical number of f.

<u>First Derivative Test</u>:
Let c be a critical number of a function f that is continuous on an open interval I containing c. If f is differentiable on the interval, except possibly at c, then $f(c)$ can be classified:

1) If $f'(x)$ changes from negative to positive at c, then $(c, \ f(c))$ is a **relative minimum** of f.
2) If $f'(x)$ changes from positive to negative at c, then $(c, \ f(c))$ is a **relative maximum** of f.

f	$\cup$	$\cap$	$\nearrow$	$\searrow$
f'	$\nearrow$	$\searrow$	$+$	$-$
f''	$+$	$-$		

<u>Second Derivative Test</u>:
Let f be a function such that the second derivative of f exists on an open interval containing c.

1) If $f'(c) = 0$ and $f''(c) > 0$, then $(c, \ f(c))$ is a relative minimum.
2) If $f'(c) = 0$ and $f''(c) < 0$, then $(c, \ f(c))$ is a relative maximum

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<u>Definition of Concavity</u>:
Let f be differentiable on an open interval I. The graph of f is **concave upward** on I if f' is increasing on the interval and **concave downward** on I if f' is decreasing on the interval.

<u>Test for Concavity</u>:
Let f be a function whose second derivative exists on an open interval I.

1) If $f''(x) > 0$ for all x in I, then the graph of f is concave upward in I.
2) If $f''(x) < 0$ for all x in I, then the graph of f is concave downward in I.

<u>Definition of an Inflection Point</u>:
A function f has an inflection point at $(c, \ f(c))$

1) if $f''(c) = 0$ or $f''(c)$ does not exist <u>and</u>
2) if f'' changes sign from positive to negative or negative to positive at $x = c$

   <u>OR</u> if $f'(x)$ changes from increasing to decreasing or decreasing to increasing at $x = c$.

First Fundamental Theorem of Calculus: $\int_a^b f'(x) dx = f(b) - f(a)$

$f(b) = f(a) + \int_a^b f'(x) dx$ final = initial + change

$f(a) = f(b) - \int_a^b f'(x) dx$ initial = final – change

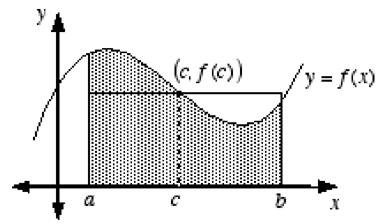
Second Fundamental Theorem of Calculus: $\dfrac{d}{dx} \int_a^x f(t) dt = f(x)$

Chain Rule Version: $\dfrac{d}{dx} \int_a^{g(x)} f(t) dt = f(g(x)) \cdot g'(x)$

Average rate of change of $f(x)$ on $[a, b]$: $\dfrac{f(b)-f(a)}{b-a}$

Average value of $f(x)$ on $[a, b]$: $f_{AVE} = \dfrac{1}{b-a}\int_a^b f(x)dx$

The rectangle has the same area as the shaded region under the curve.



Particle Motion

If an object moves along a straight line with position function $s(t)$, then its

Velocity is $v(t) = s'(t)$ **Speed** = $|v(t)|$ **Acceleration** is $a(t) = v'(t) = s''(t)$

Displacement (change in position) from $x = a$ to $x = b$ is **Displacement** = $\int_a^b v(t)\,dt$

Total Distance traveled from $x = a$ to $x = b$ is **Total Distance** = $\int_a^b |v(t)|\,dt$

At rest means $v(t) = 0$.

An object is moving left (down) when $v(t) < 0$ and an object is moving right (up) when $v(t) > 0$.

An object changes direction when velocity changes signs.
An object is speeding up when velocity and acceleration have the same sign.
An object is slowing down when velocity and acceleration have different signs.

Rate In/Rate Out

$\int_a^b (\text{rate of change})\,dt$ = amount of change from $t = a$ to $t = b$

$A(t) =$ initial amount + amount in - amount out

$A(t) =$ initial amount + $\int_0^t (\text{rate in})\,dt - \int_0^t (\text{rate out})\,dt$

$\dfrac{d}{dt}(\text{amount}) = \text{rate}$

If rate in > rate out, then amount is increasing
If rate out > rate in, then amount is decreasing
When finding abs max and/or abs max, use candidates test. To find critical values: rate in = rate out.

L'Hospital's Rule:

Suppose that f and g are differentiable functions and that $g'(x) \neq 0$ near $x = a$ and

$\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = 0$ or that $\lim_{x\to a} f(x) = \pm\infty$ and $\lim_{x\to a} g(x) = \pm\infty$

Then $\lim_{x\to a}\dfrac{f(x)}{g(x)} = \lim_{x\to a}\dfrac{f'(x)}{g'(x)}$

Integration by Parts:

Indefinite: $\int u\,dv = uv - \int v\,du$ Definite: $\int_a^b u\,dv = \left[uv - \int v\,du\right]_a^b$

<u>Order for choosing "u"</u>
L – Logarithms
I – Inverse Trig Functions
P – Polynomials
E – Exponential Functions
T – Trig Functions

Area Under a Curve

$A = \int_a^b \left[f(x) - g(x)\right] dx$ or $A = \int_a^b \left[upper - lower\right] dx$

$A = \int_c^d \left[f(y) - g(y)\right] dy$ or $A = \int_c^d \left[right - left\right] dy$

Volume

Disk Method (no hole)

A. Horizontal Axis of Rotation: $V = \pi \int_a^b (upper - lower)^2 dx$

B. Vertical Axis of Rotation: $V = \pi \int_c^d (right - left)^2\, dy$

Washer Method (with hole: whole-hole)

A. Horizontal Axis of Rotation: $V = \pi \int_a^b \left[\underbrace{(upper - lower)^2}_{whole} - \underbrace{(upper - lower)^2}_{hole}\right] dx$

B. Vertical Axis of Rotation: $V = \pi \int_c^d \left[\underbrace{(right - left)^2}_{whole} - \underbrace{(right - left)^2}_{hole}\right] dy$

Cross Sections

A. Cross sections are perpendicular to x-axis $V = \int_a^b A(x)\, dx$

B. Cross sections are perpendicular to y-axis $V = \int_c^d A(y)\, dy$

Length of a Curve

$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$ or $L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

Definition of a Taylor polynomial:

If f has n derivatives at c, then the polynomial

$$P_n(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \frac{f'''(c)}{3!}(x-c)^3 + \ldots + \frac{f^{(n)}(c)}{n!}(x-c)^n$$

is called the **nth Taylor polynomial for f at c.**

Test	Series	Converges	Diverges	Comment
Nth Term Test for Divergence	$\sum_{n=1}^{\infty} a_n$		$\lim_{n\to\infty} a_n \neq 0$	This test cannot be used to show convergence.
Geometric Series	$\sum_{n=1}^{\infty} ar^n$	$\lvert r \rvert < 1$	$\lvert r \rvert \geq 1$	If converges: $S = \frac{a_1}{1-r}$
P-Series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	$p > 1$	$p \leq 1$	
Alternate Series	$\sum_{n=1}^{\infty} (-1)^{n-1} a_n$	Converges if : 1. $a_n > 0$ 2. terms are decreasing 3. $\lim_{n\to\infty} a_n = 0$		Remainder: $\lvert R_n \rvert \leq a_{n+1}$
Ratio	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert < 1$	$\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert > 1$	No conclusion if: $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert = 1$